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G E O M E T R Y I N T H E G R A D E S

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-- T H E S I S --

For Degree of MASTER OF ARTS

In the

G R A D U A T E S C H O O L

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U N I V E R S I T Y O F I L L I N O I S

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G E O M E T R Y I N T H E G R A D E S .

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Geometry is seldom found in the school course before the years of the High School. An examination of the different curricula shows that with rare exceptions it is not found until the ninth or tenth year, and in many instances not until the twelfth. It is generally assumed that the subject is too difficult for the pupil in the grades and must be reserved till he has reached an age when he can grasp it. Geometry has a phase that belongs to the High School, for there is a phase of it that can not be presented at a more suitable time. But it also has a phase that is suited to the grades.

Every subject has a phase that is suited to the general divisions of the school, - the grades, the high-school, and the university. The root, the stem, and the leaf of the plant, its buds, flowers, and the germination of its seeds can be studied and comprehended to a degree by the pupil in the first years of his school course. He can not enter very deeply into these; he finds their functions and relations in a very general way. Later, in the High School he can see the deeper relations, organize what he observes on deeper and

further reaching fundamentals than was possible for him before. He makes a conscious effort to grasp what he gets about the fundamentals, and so forms his science of botany. In the years of the university he takes this subject and others which he has been carrying forward in the same manner, and by seeing principles which are deeper and broader than those he found in the High School, principles broader than those peculiar to any one science, he unites his sciences, - philosophizes, universalizes.

So it is with geometry. It has phases which belong to the grades, the high-school, and the university. The child can see that the angles of a triangle are equal to two right angles, by measuring them with his protractor. He can tear the angles from a paper triangle, place them so that the sides will touch, and see that the resulting angle has its sides in a straight line. He can know that the square on the hypotenuse of a right angle is equal to the sum of the squares on the legs, by drawing the squares, dividing them into units and then comparing.

In all this the learner does very little if anything with the reasons for these conditions. He simply performs with the hands, sees with the eye, and uses simple judgment in the presence of the senses. The place for the demonstration generally speaking is in the High School. In the grades he learns that the angles of a triangle are equal to two right angles. In the High School he learns why this is so, and that it must be so from the nature of thought

and reason itself,- that is sees the universality of the truth. The Pythagorean Proposition becomes full of reason. He compares the truths he gets and so gets new truths, forming his science of geometry. It is the work of the university to show him the relation of geometry to all mathematics, and to the other branches of knowledge. Between the lowest phase of perception and physical geometry which the child should begin on entering school, and the demonstrative work in the High School there is an imaginative and constructive phase proper for the intermediate grades. Rightly speaking, then, the grade work in geometry consists of primary or perceptive geometry, and intermediate or imaginative geometry; while the High school treats demonstrative geometry.

Geometry is the definite mental experience of the properties of space. Narrowing this to the first eight years of the school, geometry in the grades is the definite mental experience of the properties of space through perception and imagination, accompanied by a low form of reasoning, of consciousness.

The child enters the school with some geometrical knowledge. It is very vague and indefinite. His earliest knowledge has to do with space and time relations. In fact no experience can be except in the form of space and time. "Before the child has a clearly intellectual life on any other subject, it attains a very definite power to distinguish the square, the circle, the oval and the spiral; to recognize by appropriate likenesses of outline, even rude draw-

ings of men,dogs,horses,cats,cows,trees and various articles of furniture." Other forms of knowledge differentiate from the knowledge of space,such as arithmetic and algebra,which closely follow upon geometry. Physical,biological and psychological rest upon these. Yet when the child enters school he is generally forced to leave the form of activity which is the first he has and is not allowed to continue it until late in the school course. He will develop this activity to a certain degree in spite of the fact that he may not be led to do so. If it were a question as to whether or not geometry should be taught in the grades,the fact that the school should be in harmony with the natural activity of the child would be conclusive evidence in its favor.

The human mind is continually developing its powers. These powers,- perception,understanding and reason,are not developed uniformly,-that is the power of perception may predominate at certain times in life over that of the reason and understanding. This is true of the pupil in the grades,which gives us the right to speak of that department as grades.The predominance of the understanding and reason gives the very general divisions of the High School and the University.

During the time when perception is predominate the individual comes in contact with the outer world through the senses,-feeling,tasting,smelling,hearing and seeing. Only the most prominent attri-

butes of objects are discerned; and consequently the classification is most superficial. The deeper and more fundamental likenesses are to be seen in the High School and the University.

The mental development of the child, then, will determine the phase of geometry that must be presented to him in the grades, - only that which appeals to him through the senses and the imagination. He deals with the concrete. So the concrete phase of geometry is the phase that is suited to his years, - concrete both to the senses and imagination involving judgment.

This is illustrated in his learning that the vertical angles are equal. To him it can be a simple matter of measurement with his protractor. The form of reasoning by which it could be proved involves abstraction which is impossible for him. The area of a trapezoid can be shown by a process of reasoning, to be the product of the altitude into the line joining the middle of the sides. The child in the grades must take a paper trapezoid, and cut off the lower points as right triangles, taking care that in each case the vertical side passes through the center of the side of the trapezoid; then he must place the triangle so that the hypotenuse of each will coincide with its equal. The resulting form will be a parallelogram. He has learned the area of this figure; and it is easy enough for him to see that the base is the equivalent of the middle line and its altitude the same as that of the trapezoid. The fif-

teen or twenty demonstrations for the Pythagorean are useless to him. He needs no demonstration except that of drawing the squares, dividing them into units and comparing them. Later in the grades he can manage these truths of the trapezoid, and thus free himself from the bondage of sense perception.

Before proceeding with the actual work as it should be presented in the grades, it is necessary to note further the nature of the individual which thinks the subject.

The individual is continually trying to identify himself with the objective world. He acts on the faith that all things are unified, and he desires to become a conscious part of that unity. This "inner impulse" seeks to satisfy itself through the functions of the mind. The movement, broadly stated, is as follows:-The child is not conscious at first, of the difference between himself and his surroundings; he sees himself in his plaything, has an unconscious unity with it. Then he sees a difference between himself and his surroundings. At last he finds himself in unity with the object but the unity now is a conscious one. The individual first has an unconscious unity with the flower; then he finds a difference between the flower and himself; and, in seeking to bridge over the difference and find unity again, he studies the plant, forms his botany. Finally he discovers the truths that are common to himself and plant, reaches the unity he had in the beginning, but the unity is now deeper, richer, fuller of content and conscious.

While the movement stated is for life in general, it is also the movement for the individual act. At first the pupil does recognize the difference between himself and the triangle; then, he realizes that he and the triangle are most unlike; it is a physical object though; then, he makes the triangle a part of himself, enlarges his life by taking the triangle into it, brings about a conscious unity, for he finds it to be a thing of thought and reason.

In making the triangle a part of himself the individual must go through the process of the one who creates it. In other words he must create the triangle. The process of getting its truth must be a creative one. Wordsworth met a little cottage girl whose conversation about the future life showed that she had perfect confidence in it. Taking this theme, sublime faith, he wrote "The Little Cottage Girl", measuring every thought, every expression by its reflection of the theme. The interpreter must picture the little cottage girl, get the theme, and measure every part of the poem by its bearing upon it. He must create the poem. The inventor saw the grass falling before the scythe and conceived the idea that a rapidly moving knife would be of more service. With this in mind he made the mowing machine, judging every part by its bearing on the rapidly moving knife. The interpreter of the mowing machine must see in imagination, if not actually, the falling grass, the rapidly moving sickle, and the value of all the parts by their bearing on this central point. He must invent the mowing machine.

So it is with all learning. To know the Bartholdi Statue one must know the architect's thought; and, in doing so, become to that extent the architect. To know the human mind one must read the mind of the Creator in it, and to that extent live the life of the Creator.

So it is in learning the triangle, the cube or the sphere. One must see them in their creation, must construct them, "must imitate the Divine Intellect which has geometrized from eternity."

The individual's reliving the experience of the creator of the object makes the demonstration in geometry possible. We live in the firm faith that the ideal truth must be the same as the objective; that the triangle after all is but a mental construction; and that the geometrical theorems involved in it are but the necessities of reason. The moment there is a conflict between the ideal reason and reason in the object, the moment reason turns upon itself, we know that we are wrong and seek to set ourselves aright. To reason that the angles of a triangle are equal to three right angles, and then to find that this does not fit the triangle itself is to know that the reasoning has been wrong. The mind will not be satisfied if its conclusions are not justified by the external.

The whole movement through geometry, as through any subject, is a process of living. It is not the learning of the definition or demonstration that is given in the text. The one who does this

and nothing more has no geometry. Geometry is not the text nor is it the geometrical solid. It is the unification of the geometrical solid and the learning mind.

Geometry in the grades, then, is a process of the learning mind in its grasping the properties of space, through perception and imagination, accompanied by a low form of reason, of consciousness. The extent of this grasp is limited by the development of the learner. His direct connection with the object is through the senses. The constructive imagination enables him to see the object in its evolution.

The physical and the imaginative activity are instrumental in forcing the child to go through the constructive process necessary for him to make it a part of himself. In cutting a triangle from a card he is creating the triangle. He is forced to see it in its process of development. To have the child draw a right triangle with the squares on the three lines is to cause him to go through the same general process that the creator of that truth went through. To know geometry is to do geometry.

The means by which he can go through this activity are numerous. Anything which he can use to build his form is good, - the wooden blocks, the paper and scissors, the paper folded, the pencil and paper, tooth-picks, pegs, clay, tracing the object in space. The only law which determines the choice is that the means must be used which will give the learner the clearest conception. "The con-

structive exercises which are so prominent in the kinder-garten are admirably adapted to mathematical abstractions and generalizations." It is unfortunate that we limit these exercises to the kinder-garten to the extent we do.

Geometry in the grades is a phase of the child's way of thinking,--a way in which he makes the external world a part of himself. This unifying of self with the geometrical object is a life experience with the pupil and can be such only through his creating the object,--through his experiencing the same process that the creator experienced. The power of perception, with the creative imagination and physical activity constitutes the main channels by which the pupil is brought to experience the geometrical energy. This determines that the concrete phase of geometry shall be given.

The concrete phase of geometry as suited to the different years of the grades must now be given.

Geometry was defined as a definite mental experience of the properties of space. The pupil is to identify himself with these properties. He is to develop his geometrical sense.

The development of the subject must be accordance with certain laws which are found within the subject itself. Plans of development as given by text, teacher or committees on courses in geometry, must find their authority in the subject or they are not true to the subject. The strength of the pupil enters as a modifying factor in its unfolding.

The entire movement on the part of the pupil is from a vague conception of extension to a definite thorough understanding of it. It may be regarded as a process of analysis and synthesis in which the concept of extension is continually analyzed and reconstructed.

The solid should be used first. Length, breadth and thickness are all involved in it. The child can get his conception of plane, line and point from this and see them in their relations. Since he cannot make abstractions the object must be before him continually. "These at first are things to be felt." No effort should be made to deal with anything but the concrete at first.

The first few months, then, should be given to the various forms of the solid, - cube, sphere, cylinder, cone and pyramid. The idea, term should be given and definition should be made in every case. Definitions and illustrations should be made by the pupils. They may be superficial ones; but should be given by the pupil at

the risk of their being unscientific. They are true so far as the pupil is yet able to see; hence true for him, and true so far as they go. A statement coming from the pupil embodying his conception of the object means more to him than the correct definition which is given him by the teacher. The teacher must furnish the models of these forms and the pupil must observe them by sight and by touch. Usually models are found in school-rooms furnished by school authorities. But if this is not the case the teacher must furnish them, for they must be before the class.

Whatever the methods may be by which the teacher gets the models, the pupil must make his own forms. The means for doing this have been given. So important are they, however, in this part of the work that they may be repeated many times in connection with the development of certain points.

One of the best means is that of moulding in clay. This is interesting as well as efficient. The pupil delights to build the form; and, if he is required to build it so that the sides are smooth, the edges straight and sharp, he must take an infinite deal of pains which will develop the perceptive faculty, and skill in representing mathematical forms. The same may be said of drawing, of building with pegs, picks etc. The foundation for number work can thus be laid in the development of form. Frequent descriptions should be given. This might be regarded as part of the work in language and not form. But that does not lessen its value as a

means for fixing the form with the pupil. It is all the more valuable, because it is a source for exercises in language. This line of work is a fruitful source for descriptions all through the grades.

The models furnished by the teacher should be so selected that the pupil will see the essential characteristics of the solid and not have his attention fixed on some non-essential attributes. This requires that the models used to illustrate one form, as the cube, must be of different material, color and size. If all the cubes used were red, or, if all were made of clay, the learner might associate the color or material with the idea cube. The greater the variety with which the pupil may deal, and at the same time see the common element which gives rise to the class, the richer will be his concept cube. The generalization which he must make before he forms his definition is all the more valuable, if his mental action is the elimination of the non-essentials in a great variety, if his "cube experience" is the result of contact with all varieties of cubes.

In developing the idea solid the pupil is forced to get some idea of the plane, the line and the point. These were seen in the surface of the cube, in the meeting of the sides of the cubes and in the meeting of the edges. The next step should be a more definite conception of these.

The same models can be used that gave him his conception of

the solid. The attention must now be drawn from the attribute of solidarity and fastened on the surface only. The sides of the clay cube, the paper cone, or the pyramid, furnish plenty of illustrations. The pupil should point out these and form new surfaces.

After the idea, term, and definition of surface has been developed and defined, the difference of surfaces should be considered. The ideas, terms and definitions of plane and curved surfaces should be developed. The same should be done with face. The idea that some objects have one surface and others many should be presented.

Following the development of the idea surface is the development of edge, line, straight and curved line. Then the same with corner and point.

The pupil has now taken the first definite view of his subject matter. It has been very general. The points made must be touched many times during the succeeding terms and carried a little further each time they are met with.

During the remainder of the first year develop the straight and curved lines in different positions, with some of the simpler relations. Exercises in drawing should be given in every case. The drawing for the lower grades, especially should be the drawing of these geometrical forms. For the straight lines use the vertical, horizontal, perpendicular and oblique; the square with these three lines as diameter and diagonals; parallel lines from these three. For the curved line use the circumference of the circle;

the straight lines learned should be used as diameters.

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The work of the first year is a basis for the second, and should be reviewed. In addition develop the idea angle; the different angles, right, acute, obtuse and reflex. Exercises for these can be found in the "Drawing Tablets". Illustrations should be given by teacher and pupil in as many different ways as is necessary to make the latter's conception clear.

Then develop the planes involving the lines and angles learned. The simplest form for this is the triangle. The forms of triangles used are the right, acute and obtuse. These can be seen in the concrete as the solids were and can be formed more easily than the solids. The clay is ever useful; the scissors can cut the triangle from the paper, and so clear cut and sharp are the edges that he must be accurate. Much should be made of tracing the triangle in space, to lead the pupil to distinguish more closely between the material and the geometrical form.

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The start in the second year on planes bounded by straight lines should be continued. This requires a review of lines, and the different kinds of lines; angles, and the different kinds of angles; triangles, and the different kinds of triangles. The forms next developed should be four sided figures, circles, ellipses, and ovals. The pupil has met the square and the circle. This time

he is to see these forms involving the angles and the lines learned. Develop the idea diagonal of a square, semi-diagonal; the relations of diagonal, of semi-diagonals. Exercises should be found in "Drawing Tablets".

Develop idea of the diameter of a square, semi-diameter; relations of diameters to each other and to the sides. Develop idea of rhombus and rhomboid.

Review the circle and develop the circumference, the diameter and the radius; develop the concentric circles, semi-circle and quadrant. Develop ellipse and oval.

The pupil should have numerous exercises in drawing designs based on the circle, square, the oval and the ellipse.

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Review the point, line, plane and solid.

The pupil should define these in geometrical terms. Comparison and contrast of essential attributes should be made. The geometrical solid and the material solid should be compared and contrasted. This offers good exercises for composition and should be used a great deal.

Review horizontal, vertical, oblique, and parallel lines. In this as in all the work succeeding much should be made of comparison and contrast. Exercises in drawing these forms should be given. The "Drawing Tablets" used in this year as in the other years, should be based on the forms used in the development of geom-

etry. Review angle, right angle, acute angle and obtuse angle. Teach definition of a perpendicular line in connection with a right angle. Teach the relation of a circle to a right angle. Teach degree as applied to the circle; and the degrees into which the right angle is divided. In connection with this, exercises should be given in drawing and measuring angles. Develop the right angle.

Review triangle; right angled triangle; acute angled triangle; equilateral triangle; isosceles triangle; right angled isosceles triangle; and scalene triangle. Lead the pupil to see that the angles of a triangle are equal to two right angles. This can be done by tearing the angles from a paper triangle, placing them together and showing that a straight angle is the result. Many observations should be made, different triangles being used before conclusions are drawn.

Review quadrilateral ; parallelogram; square; diameter and diagonal of a square; rhombus; rectangle; rhomboid.

Develop the idea base and altitude. Lead the pupil to see the sum of the angles of a quadrilateral as in the triangle. Show the relation of quadrilateral and triangle.

Develop the polygon; pentagon; hexagon; and octagon.

These forms should be reproduced till the pupil is perfectly familiar with them. They are found in various combinations, some of which will be given. Other combinations may be found which are equally good. Pupils should be required to draw these from dicta-

tion and to practice till they could be produced rapidly from memory.

The Greek cross; the Latin cross; the Maltese cross; and the St. Andrew's cross: concentric squares; hexagon and six pointed star; octagon and eight pointed star: the Zigzag and Greek Fret with their varieties; new combinations of these made by the teacher or pupil; different designs made of crosses and squares; star cross; two, three and four triangles interwoven; twelve pointed star: designs on wall paper, oil-cloth, carpets, etc, which are based on familiar forms.

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While the first four years have sense perception as the determining element in the course of study, the second four years have the determining element in judgment as reached through the constructive imagination. These two stages of the course fuse together; no line can be drawn between the two. The pupil has become free from the limitations of sense perception when he has reached the second stage. He is now to gain freedom of judgment through imagination. This requires that he do much of the work without the presence of the physical form. Hence a good deal of the work given should be dictation. The judgment blends with the simpler forms of reason in the year next to the High School.

The work of the fourth year dealt with straight lines and figures bounded by straight lines. The work of the fifth deals

with curved lines and figures bounded by curved lines.

Develop the plane curve and the plane curved figure.

Review the circle and develop diameter, radius, sector, quadrant, arc, chord, and semi-circle.

Review ellipse and have it drawn by means of the string so that the pupil may see the relation of the radii to the form.

Dictate the following exercises:- quatrefoil; trefoil; moulding of circular curves; moulding of circular curves interlaced; moulding of flower forms alternately reversed; square and interlaced curves; rosette of right and curved lines; rosette, square, and circular curves; rosette, octagonal form; diaper pattern, squares and circles; square rosette of interlacing lines; borders of interlacing lines; Develop the compound curves and give exercises involving them. Require pupils to draw the Bohemian Glass Pitcher, pitchers, jugs and vases from the real objects.

"Prang's Form Study and Drawing" and "Smith's Manuals" are helpful. Most of the exercises given above are from the latter.

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The work of the previous years has been with geometrical forms constructed by teacher or pupil. The pupil should now see these forms in nature. This has been done to a greater or less degree all along the course. Especial emphasis should now be given to it. The representations have had only two dimensions; they should now

have three,models of which are found in nature and in decorations.

The work for this year,as the year preceding,consists in exercises in drawing. The law controling the selection of subjects is the same as in the fifth year,-those must be taken which best set forth the familiar forms. The following exercises are a few of the many possible ones.

Require the pupil to draw the conventional in leaf. Give practice in sketching from varieties of the natural ivy leaf.

Dictate the ivy leaf rosette. Give practice in drawing heart shaped leaves from the natural leaf.

Dictate the conventional maple leaf; maple leaf roseete in a square; triangular,hexagonal,and circular maple leaf rosette.

Dictate the lotus ornament;Greek and Moorish forms;the conventional vine leaf; acanthus leaf and acanthus rosette ;and the Mo- resque ornament.

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The pupil is now familiar with geometrical forms. This year's work should give the facts of plane geometry. Instruments are to be used for accuracy is to be added to the previous work in which speed and freedom were required.

Pupils should be required to keep work in note-book. Care should be taken that this be neat and accurate. Exercises to be used in connection with points developed should be introduced as much as possible. The first part of Olney's Geometry,Wm.G.Spen-

cer's Inventional Geometry, Hornbrook's Concrete Geometry, and Hill's First Lessons in Geometry, are helpful in this respect.

The work of the seventh and eighth years partakes more of the nature of the demonstration. This has been a characteristic of the work all through the course. More prominence is to be given from now on. The seventh year has the concrete phase of the demonstration of plane geometry; the eighth year has the concrete phase of the demonstration of solid geometry.

Review, comparing and contrasting each with the other, solid; surface; kinds of surfaces; line; kinds of lines; point.

Teach meaning of intersect and bisect, and how to bisect a line with instruments. In each case the pupil should practice until he secures speed and accuracy. Teach how to erect a perpendicular to a line at a given point in that line, and to drop a perpendicular from a point without a line to the line.

The simpler demonstrations with regard to the circle should be developed. This involves a review of his previous knowledge of the circle and the angle. The terms arc, tangent and secant are probably new and should be developed. Teach the meaning of inscribed figure; that the chord of the sixth part of the circumference of a circle is equal to the radius of the same circle; how to find the center of a circle when the circumference is drawn; how to pass the circumference through three given points and to circumscribe a triangle; that the circumference of a circle is about 3.1416

times the diameter. In the last as in other cases where measurements are taken dividers must be used.

Show how angles are generated and measured; the relative lengths of arcs and from that the construction of equal angles; that an angle is measured by an arc of a circle when the center of the circle and the vertex of the angle are the same; that an inscribed angle is equal to one half the arc included between the sides; the bisecting of an angle. By means of the circumference show that opposite angles are equal. Show how to inscribe an equilateral triangle.

Repeat the finding of the sum of the angles of a triangle by tearing off the angles of a paper triangle and placing them together. Show the same thing by the inscribed triangle. Show how to inscribe a circle within a triangle. Require pupils to bisect the angles of a triangle; to draw lines from the vertices to the middle of the opposite sides; to draw perpendiculars to the sides from the vertices, and note results.

Teach meaning of similar figures, equivalent figures, and equal figures.

Review the quadrilaterals. Teach how to construct a square and how to find its area. Teach how to construct a rectangle and to find its area.

Show the relation of any parallelogram to a rectangle of the same base and altitude. Show the relation of a triangle to a parallelogram and how to find its area.

Show relation of a trapezoid to a parallelogram, and how to find the area of a trapezoid.

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The work of this year is in solid geometry and will complete the first circle for the pupil in his movement through the subject. He began with the solid and will end with it. The drawings will be more difficult, as will the other ways of constructing the object. The note-book is still used and should be subject to the same requirements for neatness and accuracy that were made during the seventh year. Practice for speed and accuracy should be required in every case, and numerous problems bearing on the different points developed.

Review the definition of plane, reemphasizing the fact that the plane is the product of a moving line. The plane must be seen as a number of lines. Review parallel lines. Have pupils show the conditions under which a line is parallel to a plane. Compare and contrast parallel lines and parallel line and plane.

Develop foot of a line. Review line perpendicular to another line. Have pupil show under what conditions a line is perpendicular to a plane. Have him compare and contrast perpendicular lines and perpendicular line and plane.

Have pupil to show under what conditions planes are parallel, and to compare parallel planes with parallel lines.

Develop dihedral angle; faces, edge and plane angle of dihedral angle; Have pupils give all likenesses and differences between a plane angle and a dihedral angle.

Have pupil show under what conditions two planes are perpendicular to each other.

Review the solid; the cube; the faces and edge of a cube.

Develop the idea volume. Have pupil compare and contrast geometrical with physical solid. Have pupil find the area of a cube.

A number of small blocks should be used. These will be the units of the large cube and can be easily counted while the cube is being built. The pupil should see the relation of this volume to one edge.

The volume of a rectangle solid should be found in the same way. The volume should be found in reference to the edges. Likenesses and differences between this form and the cube should be shown. In both cases the pupil should show how the process of finding the volume grows out of the fact that the solid is the result of a moving plane.

The definition and terms of a prism should be developed. The pupil must point out differences and likenesses between this and the cube and the rectangle. The right prism should be used first because the movement from the forms just studied will be easier. He should see how the volume is found in the light of the volume just learned.

The rectangular prism might be built up as the cube was. The pupil should not need these before him as a physical form at all times. By this time he should be able to hold them before him in thought.

The pupil should move next to oblique prism, compare and contrast these with the right prism, and be forced to determine volume.

The triangular, hexagonal and other forms of the prism should be developed during the development of the rectangular and oblique forms.

The cylinder should be developed as the other forms were. The pupil should develop this, and, perhaps some of the previous ones with little or no guidance from the teacher. The movement is the same for all and he should now be familiar with this movement. The pupil should show by the form that the cylinder is generated by a rectangle revolving about one of its sides, or a circle moving in a straight line.

The pyramid, cone and sphere with terms belonging to each should be developed as in other solids.

This should complete the movement for the grades. It began with the solid and ends with it. The first cycle is complete. Starting with the handling and seeing of the physical form he learns to see it without having it held before him.

The superficial relations which he establishes in the beginning be-
deeper till he rounds up with the eighth grade work where reason
should begin to be the prominent factor.

The above course which is intended to be suggestive in its
details, properly planned and executed, must give to the pupil what
was planned in the beginning - the idea of geometrical solid with
its relations from the concrete side. His first concept was most
vague and indefinite; the last should be clear and definite.
The movement through the simpler forms of reason which characteri-
zed the seventh and eighth grades continues into deeper phases into
the high school, where its work should be strictly logical and test-
ed by the laws of reason. At the end of the high school course
the concept of the solid should be fuller and more definite than at
the end of the grade work.

The movement may seem slow for the one who would gain a fact
that it might return to him immediately. The one who learns the
triangle simply that he may earn the dollar by surveying the field
would be dissatisfied. But such an one has not learned the tri-
angle. He has simply gotten an isolated fact. If he follows
it in its development he goes through the fundamental movement of
all education; he observes, compares and contrasts, generalizes and
defines. The frequent exercises that have been given to firmly
fix a point will cover all that the materialist can desire.

The one gives the other and more besides. One gives the triangle and moves into other forms; the other touches the triangle only.

The discipline which he has received has made him accurate, thorough and methodical in his observations. The imagination has been brought under control so that the image can be pictured clear out and seen in its construction. The work has been carried alongside of other grade work much of which has been supplementary to it. The different forms have furnished the very best material for descriptions in the language work, and these descriptions have caused closer, and more accurate observation on the part of the pupil. The drawings, clay models, paper cuttings and foldings have all served the same purpose. The correct habits which he forms here along with those which he must have formed in his work in other subjects are necessary for full intellectual life. "He lives in a world of forms and the mastery of them is conditioned on thorough and systematic observation."

The ultimate problem of the intellect is to grasp the separate phenomena of the universe into unity. When the pupil grasps the unity in the class triangle or rectangle he has taken one step towards the solution of the intellect's problem.

In addition to the intellectual side geometry appeals to the emotional. Ordinarily geometry is regarded as appealing to the intellect only. The intellect may be paramount, but there is al-

ways emotion arising from the union of self with the exterior.

This "flash of agreement" creates an intellectual delight.

"The strength of the emotion aroused is a fairly true index of the clearness, distinctness, and firmness of the intellectual activity grasping the unity of the objects."

It will be remembered it was said that "the inner impulse" of the soul is to identify itself with the exterior. The result of this union is aesthetical, whether it be in literature where we do not question it, or in mathematics where we do question it. The delight which one experiences when a hard proposition is solved or when the common attribute of all pyramids is grasped is as truly an aesthetical experience as that which belongs to the reading of "The Rainy Day" or "The Vision of Sir Launfal".

Besides the intellect and sensibility geometry appeals to the will. This is, perhaps, as hidden as its appeal to the sensibilities. The appeal is easily seen in the oration, but discerned with some difficulty in the learning of a geometrical fact.

Yet the mastery of a single truth changes the whole tenor of the learner's life. "The pupil has had an enjoyment of truth; and to that extent has become a truth lover, and, therefore, has a tendency to become a truth seeker."

Every lesson has to do with "the tendency to seek truth, beauty and virtue." The lesson in geometry is a part of life's up-

building as the sermon or the lesson on ethics,- it is the sermon, the ethics itself. When the pupil feels his way through a subject and touches the truth in it, his must assume a new attitude towards his ideals, and we call that living. When the pupil thinks the triangle he is doing the same thing, he is living so much of life. "He lives in his work not by it."